21.01.2022

M.Sc. Sem III & IV

ADVANCED TOPOLOGY

Compact spaces

Open cover: Let X be a topo. space with topology T.

By open cover $\{U_\alpha\}$ of X, we mean a collection $\{U_\alpha\} \subseteq T$ such that

$$X \subseteq \bigcup_\alpha U_\alpha.$$

Finite subcover: By finite subcover of an open cover $\{U_\alpha\}$ we mean finite subset $\{U_{\alpha_1}, U_{\alpha_2}, \ldots U_{\alpha_k}\}$ of $\{U_\alpha\}$ such that

$$X \subseteq U_{\alpha_1} \cup U_{\alpha_2} \cup U_{\alpha_3} \ldots \cup U_{\alpha_k}.$$

Compact space: A topological space (X, T) is compact if any open cover of X admits a finite subcover.

T_2 - Space (Hausdorff space)

~~If is~~ A topological space (X, T) is said to be a T_2 - space if for any $x, y \in X$, $x \neq y$ there exist open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \phi$.

i.e. A Hausdorff space is a topological space in which each pair of distinct points can be separated by a disjoint open set.

Example Let $X = \{1, 2, 3\}$

Let $T = \{\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, X, \phi\}$

Now $1, 2 \in X$, $1 \neq 2$ and $1 \in \{1\}$, $2 \in \{2\}$ and

$\{1\} \cap \{2\} = \phi$

Also, $2, 3 \in X$, $2 \neq 3$ and $2 \in \{2\}$, $3 \in \{3\}$ and

$\{2\} \cap \{3\} = \phi$.

So, X is a Hausdorff Space.